On Achievable Schemes of Interference Alignment with Double-Layered Symbol Extensions in Interference Channel

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Abstract

This paper looks into the K-user interference channel. Interference Alignment is much likely to be applied with double-layered symbol extensions, either for constant channels in the Høst-Madsen-Nosratinia conjecture or slowly changing channels. In our work, the core idea relies on double-layered symbol extensions to artificially construct equivalent time-variant channels to provide crucial *channel randomness or relativity* required by conventional Cadambe-Jafar scheme in time-variant channels [1].

I. Introduction

The Høst-Madsen-Nosratinia conjecture was proposed in the investigation on the multiplexing gain of a network with K source nodes and K destination nodes in pairs while each node has only a single antenna and all the nodes could cooperate [2]. The multiplexing gain is also known as degrees of freedom (DoF) of the channel, in particular denoting the pre-log factor of the rate as signal-noise-ration (SNR) approaches infinity. So the question is: how large a multiplexing gain/DoF is possible for the K-pair network. It is proved that for $K = 2$ the DoF is 1. While for the general K-pair network, it is proved that the upperbound of achievable DoF is $K/2$, i.e. the network obtains at most $K/2$ DoF. However, it is believed that $K/2$ is not a tight upperbound and furthermore it is conjectured that the achievable DoF is still only one.

It is important to highlight an implicit condition in the Høst-Madsen-Nosratinia conjecture, i.e all the channels are constant. So that it makes difference with the prominent result of conventional Cadambe-Jafar scheme of interference alignment (IA) as in [1], [3]. Cadambe-Jafar scheme claims that, on condition the channels are time-variant, the K-pair network is able to approach $K/2$ DoF. If the channels are constant, Cadambe-Jafar does not work due to its loss of *channel randomness or relativity* of the network, which we discuss in detail in following sections.

Later, the Høst-Madsen-Nosratinia conjecture was first settled by Cadambe, Jafar and Wang as in [4]. It shows that at least 1.2 DoF are achievable on the complex Gaussian 3-user interference channel with constant coefficients for almost all values except for a subset of measure zero. A novel idea of asymmetric complex signaling is proposed, in which the inputs are chosen to be complex but not circularly symmetric.

In our work, a novel method is proposed for K-user interference network with constant channels. The rest of this paper is organized as follows. In Section II, the system model and preliminaries are introduced for the K-user network. In Section III, the novel design of double-layered symbol extensions are proposed. In Section IV, numerical results and performance are given. In Section V, conclusions and remarks are made.

II. System Model and Preliminaries

Define all users in a set $\mathcal{K} = \{1, 2, \cdots, K\}$. Denote the channel from j-th source to k-th destination as a complex scalar h_{kj}. Specifically at t-th time slot, the channel is denoted as $\mathrm{h}_{kj}^{[t]}$. Use symbol extensions for the design, so define the length of extensions as T, i.e. the dimension of the extended signal vector. Correspondingly, the effective channel from j-th source to k-th destination is denoted as the following equation:

$$\mathbf{H}_{kj} = \mathrm{Diag}\{\mathrm{h}_{kj}^{[1]}, \mathrm{h}_{kj}^{[2]}, \cdots, \mathrm{h}_{kj}^{[T]}\} \tag{1}$$

in which $\mathrm{Diag}\{\cdot\}$ represents a diagonal matrix composed of diagonal elements of the scalar channel coefficients at all T time slots so that the dimension is set as $\mathbf{H}_{kj} \in \mathbb{C}^{T\times T}$.

Let each user transmit d_k datastreams, and set each precoder $\mathbf{V}_k \in \mathbb{C}^{T\times d_k}$ respectively. Then the DoF could be calculated as $(\frac{d_1}{T}, \frac{d_2}{T}, \ldots, \frac{d_K}{T})$ for all K users and the total DoF of network is $\frac{(d_1+d_2+\ldots+d_K)}{T}$.

A. Conventional Interference Alignment

According to Cadambe-Jafar scheme in [1], [3], interference alignment is implemented by setting the following condition:

$$\begin{aligned} &\mathbf{H}_{1i}\mathbf{V}_i = \mathbf{H}_{13}\mathbf{V}_3, \quad \forall i \in \mathcal{K}\backslash\{1,3\} \\ &\mathbf{H}_{jk}\mathbf{V}_k \prec \mathbf{H}_{j1}\mathbf{V}_1, \quad \forall j \in \mathcal{K}\backslash\{1,k\}, \forall k \in \mathcal{K}\backslash\{1\} \end{aligned} \tag{2}$$

Then the solution of precoders satisfying the condition (2) is given by:

$$\mathbf{V}_1 = \left\{ \left(\prod_{k,l\in\mathcal{K}\backslash\{1\}, k\neq l, (k,l)\neq(2,3)} (\mathbf{T}_{kl})^{n_{kl}} \right) \cdot \mathbf{w} \middle| \forall n_{kl} \leq n \right\} \tag{3}$$

$$\mathbf{V}_3 = \left\{ \mathbf{H}_{21}(\mathbf{H}_{23})^{-1} \left(\prod_{k,l\in\mathcal{K}\backslash\{1\}, k\neq l, (k,l)\neq(2,3)} (\mathbf{T}_{kl})^{n_{kl}} \right) \cdot \mathbf{w} \middle| \forall n_{kl} \leq n-1 \right\} \tag{4}$$

$$\mathbf{V}_i = (\mathbf{H}_{1i})^{-1}\mathbf{H}_{13} \cdot \mathbf{V}_3, \quad \forall i \in \mathcal{K}\backslash\{1,3\} \tag{5}$$

in which

$$\begin{aligned} &\mathbf{T}_{kl} = \mathbf{H}_{21}(\mathbf{H}_{23})^{-1}\mathbf{H}_{13}(\mathbf{H}_{k1})^{-1}\mathbf{H}_{kl}(\mathbf{H}_{1l})^{-1} \ \forall l,k \in \mathcal{K}\backslash\{1\} \\ &\mathbf{w} = [1 \ 1 \cdots \ 1]^T \in \mathbb{C}^{M\times 1} \end{aligned} \tag{6}$$

In the solution of (3), (4) and (5), the length of symbol extensions is set as $T = (n+1)^N + n^N$ where $n \in \mathbb{N}$ and $N = (K-1)(K-2)-1$. Then the precoders have different dimensions: $\mathbf{V}_1 \in \mathbb{C}^{M\times(n+1)^N}$ and $\mathbf{V}_j \in \mathbb{C}^{M\times n^N}, \forall j \in \mathcal{K}\backslash\{1\}$. So that the first user obtains $\frac{(n+1)^N}{(n+1)^N+n^N}$ DoF and all the other $(K-1)$ users obtain $\frac{(n)^N}{(n+1)^N+n^N}$ DoF for each respectively. When $n\to\infty$, the obtained DoF for each user approaches $1/2$, and the total DoF for the K-pair network approaches $K/2$.

B. Constant Channel Issue and Slowly Changing Channel Issue

When the channel is constant, then the effective channel of $\mathbf{H}_{kj}$ in (1) becomes:

$$\begin{aligned} &\mathbf{H}_{kj} = \mathrm{h}_{kj}\mathbf{I}_T \\ &\mathrm{h}_{kj}^{[1]} = \mathrm{h}_{kj}^{[2]} = \cdots = \mathrm{h}_{kj}^{[T]} = \mathrm{h}_{kj} \end{aligned} \tag{7}$$

where $\mathbf{I}_T$ is a T-dimensional identity matrix.

Then the intermediate matrix $\mathbf{T}_{kl}$ in (6) is calculated as $(\mathrm{h}_{21}\mathrm{h}_{23}^{-1}\mathrm{h}_{13}\mathrm{h}_{k1}^{-1}\mathrm{h}_{kl}\mathrm{h}_{1l}^{-1})\mathbf{I}_T$. Obviously, the precoders $\mathbf{V}_1$ in (3) and $\mathbf{V}_3$ in (4) are composed of linear dependent columns respectively, and so is $\mathbf{V}_i$ as in (5). So that the precoding schemes are not applicable when the channels are constant. The reason is the K-pair network loses *channel randomness or relativity*, which is required for interference alignment as in [1].

When the channel is slowly changing as in most of realistic situations, it is necessary to wait for much longer time to combine all the required number of T time slots to apply IA scheme. The delay is not tolerable in practice. As a primitive investigation, we propose a simplified model for slowly changing channels as following:

$$\begin{aligned} \mathrm{h}_{kj}^{[1]} = \mathrm{h}_{kj}^{[2]} = \cdots = \mathrm{h}_{kj}^{[T/2]} = \mathrm{h}_{kj}^{\star} \\ \mathrm{h}_{kj}^{[T/2+1]} = \mathrm{h}_{kj}^{[T/2+2]} = \cdots = \mathrm{h}_{kj}^{[T]} = \mathrm{h}_{kj}^{\circ} \end{aligned} \tag{8}$$

III. Proposed Scheme based on Symbol Extensions

First, observe and analyze the above constant channel issue in (7), and naturally come up with a idea of artificially fluctuating the symbol extensions of channels to produce randomness. The procedure is as follows.

A. Unsuccessful Trial: Natural and Naive Fluctuation Coding

The natural and naive method is to fluctuate the coding at all nodes with an additional gain on purpose to construct effective time-variant channels (not successful although). Let the j-th source node has a gain of $\alpha_j^{[t]}$ at the time slot t, which is randomly drawn from a continuous distribution; and the k-th destination node has a gain of $\beta_k^{[t]}$ at the time slot t, which is also randomly drawn from a continuous distribution. Then the equivalent channel is denoted as $\tilde{\mathrm{h}}_{kj}^{[t]}$ in the following equation:

$$\tilde{\mathrm{h}}_{kj}^{[t]} = \beta_k^{[t]}\mathrm{h}_{kj}^{[t]}\alpha_j^{[t]} \tag{9}$$

Then observe the time-extended effective channel $\mathbf{H}_{kj}$ in (1) becomes:

$$\begin{aligned} \mathbf{H}_{kj} &= \mathrm{Diag}\{\tilde{\mathrm{h}}_{kj}^{[1]}, \tilde{\mathrm{h}}_{kj}^{[2]}, \cdots, \tilde{\mathrm{h}}_{kj}^{[T]}\} \\ &= \mathrm{Diag}\{\beta_k^{[1]}\mathrm{h}_{kj}^{[1]}\alpha_j^{[1]}, \beta_k^{[2]}\mathrm{h}_{kj}^{[2]}\alpha_j^{[2]}, \cdots, \beta_k^{[T]}\mathrm{h}_{kj}^{[T]}\alpha_j^{[T]}\} \end{aligned} \tag{10}$$

It could be further decomposed into a concise and explicit form:

$$
\begin{aligned}
\mathbf{H}_{kj} &= \Xi_k \Delta_{kj} \Omega_j \\
\Xi_k &= \text{Diag}\{\beta_k^{[1]}, \beta_k^{[2]}, \cdots, \beta_k^{[T]}\} \\
\Delta_{kj} &= \text{Diag}\{\text{h}_{kj}^{[1]}, \text{h}_{kj}^{[2]}, \cdots, \text{h}_{kj}^{[T]}\} \\
\Omega_j &= \text{Diag}\{\alpha_j^{[1]}, \alpha_j^{[2]}, \cdots, \alpha_j^{[T]}\}
\end{aligned}
\tag{11}
$$

When the channel is constant, the time-extended effective channel $\mathbf{H}_{kj}$ in (7) becomes:

$$
\begin{aligned}
\mathbf{H}_{kj} &= \text{Diag}\{\tilde{\text{h}}_{kj}^{[1]}, \tilde{\text{h}}_{kj}^{[2]}, \cdots, \tilde{\text{h}}_{kj}^{[T]}\} \\
&= \text{h}_{kj} \cdot \text{Diag}\{\beta_k^{[1]}\alpha_j^{[1]}, \beta_k^{[2]}\alpha_j^{[2]}, \cdots, \beta_k^{[T]}\alpha_j^{[T]}\}
\end{aligned}
\tag{12}
$$

So that in the case of constant channel, observe that $\mathbf{H}_{kj}$ in (12) is indeed an effective time-variant channel. However, to unveil its real impact for the complete scheme, it is further decomposed with the following equation:

$$
\begin{aligned}
\mathbf{H}_{kj} &= \text{h}_{kj} \cdot \Xi_k \Omega_j \\
\Xi_k &= \text{Diag}\{\beta_k^{[1]}, \beta_k^{[2]}, \cdots, \beta_k^{[T]}\} \\
\Omega_j &= \text{Diag}\{\alpha_j^{[1]}, \alpha_j^{[2]}, \cdots, \alpha_j^{[T]}\}
\end{aligned}
\tag{13}
$$

Then check the intermediate matrix $\mathbf{T}_{kl}$ in (6) again, with the surprising result as following:

$$
\begin{aligned}
\mathbf{T}_{kl} &= \mathbf{H}_{21}(\mathbf{H}_{23})^{-1}\mathbf{H}_{13}(\mathbf{H}_{k1})^{-1}\mathbf{H}_{kl}(\mathbf{H}_{1l})^{-1} \\
&= \text{h}_{21}\Xi_2\Omega_1(\text{h}_{23}\Xi_2\Omega_3)^{-1}\text{h}_{13}\Xi_1\Omega_3(\text{h}_{k1}\Xi_k\Omega_1)^{-1}\text{h}_{kl}\Xi_k\Omega_l(\text{h}_{1l}\Xi_1\Omega_l)^{-1} \\
&= \text{h}_{21}(\text{h}_{23})^{-1}\text{h}_{13}(\text{h}_{k1})^{-1}\text{h}_{kl}(\text{h}_{1l})^{-1} \cdot \mathbf{I}_T
\end{aligned}
\tag{14}
$$

Surprisingly in (14), $\mathbf{T}_{kl}$ is still a scaled identity matrix. So that the precoders $\mathbf{V}_1$ in (3), $\mathbf{V}_3$ in (4), and $\mathbf{V}_i$ in (5) degenerate to matrices with linear dependent columns. In conclusion, the method with naive fluctuation symbol extensions is unsuccessful to create channel randomness in constant channels to apply Cadambe-Jafar scheme to achieve interference alignment.

While for slow changing channels as in the same expression of (10), all the statuses has a relationship as in (8). It needs further validation and proof whether IA scheme is applicable.

B. Novel Design: Double-Layered Symbol Extensions

The naive fluctuation coding to construct time-variant channels in (9) and (12) is proved to be not successful according to (14). However, it still inspires important clues for a novel achievable design. This novel method is called double-layered symbol extensions.

For the sake of simplicity, let T be an even number. Divide all T time slots into two pieces, i.e. from 1 to $T/2$ as the first piece, and from $(T/2+1)$ to T as the second piece. Pair each two time slots in each piece respectively in sequence, i.e. 1 and $(T/2+1)$, and 2 and $(T/2+2)$ etc. It is equivalent to add two statuses of channels to form one virtual status of the network. So there are totally $T/2$ statuses, and the (t)-th virtual channel status is obtained from the t-th time slot and $(T/2+t)$-th time slot. In particular, $\tilde{\text{h}}_{kj}^{t}$ is constructed as following:

$$
\tilde{\text{h}}_{kj}^{[t]} = \beta_k^{[t]}\text{h}_{kj}^{[t]}\alpha_j^{[t]} + \beta_k^{[T/2+t]}\text{h}_{kj}^{[T/2+t]}\alpha_j^{[T/2+t]}
\tag{15}
$$

To implement the design of (3), (4) and (5), the length of symbol extensions is set as $T = 2[(n+1)^N + n^N]$ where $n \in \mathbb{N}$ and $N = (K-1)(K-2)-1$. Then the time-extended effective channel $\mathbf{H}_{kj}$ in (12) is updated and replaced by the following equation:

$$
\begin{aligned}
\mathbf{H}_{kj} &= \text{Diag}\{\tilde{\text{h}}_{kj}^{[1]}, \tilde{\text{h}}_{kj}^{[2]}, \cdots, \tilde{\text{h}}_{kj}^{[T/2]}\} \\
&= \text{Diag}\{\beta_k^{[1]}\text{h}_{kj}^{[1]}\alpha_j^{[1]} + \beta_k^{[T/2+1]}\text{h}_{kj}^{[T/2+1]}\alpha_j^{[T/2+1]}, \beta_k^{[2]}\text{h}_{kj}^{[2]}\alpha_j^{[2]} + \beta_k^{[T/2+2]}\text{h}_{kj}^{[T/2+2]}\alpha_j^{[T/2+2]}, \cdots \\
&\qquad , \beta_k^{[T/2]}\text{h}_{kj}^{[T/2]}\alpha_j^{[T/2]} + \beta_k^{[T]}\text{h}_{kj}^{[T]}\alpha_j^{[T]}\}
\end{aligned}
\tag{16}
$$

Observe that $\mathbf{H}_{kj}$ in (16) is effectively time-variant and has the new dimension $\mathbf{H}_{kj} \in \mathbb{C}^{(T/2)\times(T/2)}$. It could be further decomposed into the following equation:

$$
\begin{aligned}
&\mathbf{H}_{kj} = \Xi_k^{\star}\Delta_{kj}^{\star}\Omega_j^{\star} + \cdot\Xi_k^{\circ}\Delta_{kj}^{\circ}\Omega_j^{\circ} \\
&\Xi_k^{\star} = \text{Diag}\{\beta_k^{[1]}, \beta_k^{[2]}, \cdots, \beta_k^{[T/2]}\}, \Xi_k^{\circ} = \text{Diag}\{\beta_k^{[T/2+1]}, \beta_k^{[T/2+2]}, \cdots, \beta_k^{[T]}\} \\
&\Delta_{kj}^{\star} = \text{Diag}\{\text{h}_{kj}^{[1]}, \text{h}_{kj}^{[2]}, \cdots, \text{h}_{kj}^{[T/2]}\}, \Delta_{kj}^{\circ} = \text{Diag}\{\text{h}_{kj}^{[T/2+1]}, \text{h}_{kj}^{[T/2+2]}, \cdots, \text{h}_{kj}^{[T]}\} \\
&\Omega_j^{\star} = \text{Diag}\{\alpha_j^{[1]}, \alpha_j^{[2]}, \cdots, \alpha_j^{[T/2]}\}, \Omega_j^{\circ} = \text{Diag}\{\alpha_j^{[T/2+1]}, \alpha_j^{[T/2+2]}, \cdots, \alpha_j^{[T]}\} \\
&\Xi_k^{\star},\ \Xi_k^{\circ},\ \Delta_{kj}^{\star},\ \Delta_{kj}^{\circ},\ \Omega_j^{\star},\ \Omega_j^{\circ} \in \mathbb{C}^{(T/2)\times(T/2)}
\end{aligned}
\tag{17}
$$

Then check the intermediate matrix $\mathbf{T}_{kl}$ in (6) again as following:

$$\begin{aligned}\mathbf{T}_{kl} &= \mathbf{H}_{21}(\mathbf{H}_{23})^{-1}\mathbf{H}_{13}(\mathbf{H}_{k1})^{-1}\mathbf{H}_{kl}(\mathbf{H}_{1l})^{-1} \\ &= (\Xi_2^\star\Delta_{21}^\star\Omega_1^\star + \Xi_2^\circ\Delta_{21}^\circ\Omega_1^\circ) \\ &\quad (\Xi_2^\star\Delta_{23}^\star\Omega_3^\star + \Xi_2^\circ\Delta_{23}^\circ\Omega_3^\circ)^{-1}(\Xi_1^\star\Delta_{13}^\star\Omega_3^\star + \Xi_1^\circ\Delta_{13}^\circ\Omega_3^\circ)(\Xi_k^\star\Delta_{k1}^\star\Omega_1^\star + \Xi_k^\circ\Delta_{k1}^\circ\Omega_1^\circ)^{-1} \\ &\quad (\Xi_k^\star\Delta_{kl}^\star\Omega_l^\star + \Xi_k^\circ\Delta_{kl}^\circ\Omega_l^\circ)(\Xi_1^\star\Delta_{1l}^\star\Omega_l^\star + \Xi_1^\circ\Delta_{1l}^\circ\Omega_l^\circ)^{-1}\end{aligned} \tag{18}$$

Based on the key theorem in Cadambe-Jafar scheme [1, Theorem 1], we look into the K-pair single-antenna network in constant channels as in (7) and (13), and slowly changing channels as in (8) and (11). If effective channels are constructed with double-layered symbol extensions as in (16) and (17), then conventional Cadambe-Jafar scheme in (3), (4) and (5) could be much likely to be applied on the effective channels to approach $K/4$ DoF for the network. The detail achievable scheme is provided in *Theorem 1* in [1, Theorem 1], in which [1, Section IV, Subsection B] dealt with the 3-user case and then [1, Appendix III] coped with the arbitrary K-user case.

First, consider the alignment/overlapping condition of interference subspaces of (2). It is easily verified that the design of precoders in (3), (4) and (5) satisfy the alignment condition. It does not require any special features of $\mathbf{H}_{kj}$ or $\mathbf{T}_{kl}$, so that the double-layered symbol extensions do not impact the IA scheme in terms of (17) and (18).

Second, it is necessary to verify that the desired signals are composed of linearly independent streams and at the same time they are linearly independent of the interferences so that the streams could be decoded by zero-forcing the interference.

Without losing generality, only take the received signal vectors at the 1-st receiver as an example: $\mathbf{R} = [\mathbf{H}_{11}\mathbf{V}_1 \ \mathbf{H}_{12}\mathbf{V}_2]$. Notice the dimension is set as $\mathbf{R} \in \mathbb{C}^{(T/2)\times(T/2)}$. As mentioned, $\mathbf{H}_{12}\mathbf{V}_2$ represents all the aligned interference subspaces from different transmitters to the 1-st receiver. Therefore, in order to prove $\mathbf{H}_{11}\mathbf{V}_1$ has full linearly independent columns, it only needs to show $\mathbf{R}$ has full linearly independent columns, i.e. the matrix $\mathbf{R}$ has full rank of $T/2$.

Transform $\mathbf{R}$ to an equivalent matrix $\mathbf{S} = [\mathbf{V}_1 \ (\mathbf{H}_{11})^{-1}\mathbf{H}_{12}\mathbf{V}_2]$. In detail, it is composed of $(n+1)^N$ columns in the form of $\prod \mathbf{T}_{kl}^{n_{kl}}\mathbf{w}$ and n^N columns in the form of $(\mathbf{H}_{11})^{-1}\mathbf{H}_{12}\prod \mathbf{T}_{kl}^{n_{kl}}\mathbf{w}$. Let the diagonal entries of $\mathbf{T}_{kl}$ be $\lambda_{kl}^{\langle 1\rangle}, \lambda_{kl}^{\langle 2\rangle}, \dots, \lambda_{kl}^{\langle T/2\rangle}$ and the diagonal entries of $(\mathbf{H}_{11})^{-1}\mathbf{H}_{12}$ be $\kappa^{\langle 1\rangle}, \kappa^{\langle 2\rangle}, \dots, \kappa^{\langle T/2\rangle}$. So that the q-th entry is obtained from q-th and $(T/2+q)$-th time slots. Let $s = q$, $t = T/2 + q$, then $\lambda_{kl}^{\langle q\rangle}$ and $\kappa^{\langle q\rangle}$ are presented as:

$$\begin{aligned}&\lambda_{kl}^{\langle q\rangle} = \\ &\frac{(\beta_2^{[s]}\mathrm{h}_{21}^{[s]}\alpha_1^{[s]} + \beta_2^{[t]}\mathrm{h}_{21}^{[t]}\alpha_1^{[t]})(\beta_1^{[s]}\mathrm{h}_{13}^{[s]}\alpha_3^{[s]} + \beta_1^{[t]}\mathrm{h}_{13}^{[t]}\alpha_3^{[t]})(\beta_k^{[s]}\mathrm{h}_{kl}^{[s]}\alpha_l^{[s]} + \beta_k^{[t]}\mathrm{h}_{kl}^{[t]}\alpha_l^{[t]})}{(\beta_2^{[s]}\mathrm{h}_{23}^{[s]}\alpha_3^{[s]} + \beta_2^{[t]}\mathrm{h}_{23}^{[t]}\alpha_3^{[t]})(\beta_k^{[s]}\mathrm{h}_{k1}^{[s]}\alpha_1^{[s]} + \beta_k^{[t]}\mathrm{h}_{k1}^{[t]}\alpha_1^{[t]})(\beta_1^{[s]}\mathrm{h}_{1l}^{[s]}\alpha_l^{[s]} + \beta_1^{[t]}\mathrm{h}_{1l}^{[t]}\alpha_l^{[t]})}\end{aligned} \tag{19}$$

$$\kappa^{\langle q\rangle} = \frac{(\beta_1^{[s]}\mathrm{h}_{12}^{[s]}\alpha_2^{[s]} + \beta_1^{[t]}\mathrm{h}_{12}^{[t]}\alpha_2^{[t]})}{(\beta_1^{[s]}\mathrm{h}_{11}^{[s]}\alpha_1^{[s]} + \beta_1^{[t]}\mathrm{h}_{11}^{[t]}\alpha_1^{[t]})} \tag{20}$$

Then the matrix $\mathbf{S}$ is composed of elements of $\lambda_{kl}^{\langle q\rangle}$ and $\kappa^{\langle q\rangle}$. As mentioned in the beginning, the scheme is based on the same procedure in [1, Section IV, Subsection B] and [1, Appendix III]. The detail is not repeatedly described here.

To apply the achievable scheme to our case of either constant channel in (7) or slowly changing channel in (8), two fundamental conditions are required. 1) notice a key requirement is that $\kappa^{\langle q\rangle}$ is a random variable drawn from a continuous distribution so that it has probability zero to take a value of the corresponding linear equations. 2) Furthermore, notice another key requirement that all $\lambda_{kl}^{\langle q\rangle}$ are drawn independently from a continuous distribution and they are all distinct almost surely so that they have probability zero to be equal to the roots of corresponding finite degree polynomials.

For our case, check (19) and (20) which guarantee that $\lambda_{kl}^{\langle q\rangle}$ and $\kappa^{\langle q\rangle}$ are random values since the gains $\alpha_i^{[t]}$ and $\beta_j^{[t]}$ are randomly generated from continuous distributions. It is also obvious that they are independent because each distinct q-th entry only uses variables within the corresponding two time slots. Finally, check $\lambda_{kl}^{\langle q_1\rangle}$ and $\lambda_{kl}^{\langle q_2\rangle}$ for $q_1 \neq q_2$, and it is obvious they are distinct so that it prevents the failure of (14) as in the case of naive fluctuation coding. On condition the above randomness of $\lambda_{kl}^{\langle q\rangle}$ and $\kappa^{\langle q\rangle}$ is guaranteed, it is possible to proceed the IA scheme. However, it still needs further validation and rigorous proof to check linear independency of all high-rank exponentials in the constructed signal space.

IV. Primitive Numerical Results

To further validate the proposed novel method of double-layered symbol extensions, numerical results are given as well. First, we look at a 3-user network applying Cadambe-Jafar scheme of interference alignment as in (3), (4), and (5). For comparison, on one hand, we use the natural and naive fluctuation coding as in (9) and (12); on the other hand we use the double-layered symbol extensions as in (15) and (16), both in constant channels. These two cases are shown in Fig. 1.

In Fig. 1, since it is a 3-user network, i.e. $K = 3$, then the power component $N = (K-1)(K-2)-1 = 1$. For the natural and naive fluctuation coding, set the length of symbol extensions $T = 2n+1$ and $n = 2$, so that the network is supposed to obtain a total DoF of $\frac{3n+1}{2n+1} = 7/5$ if Cadambe-Jafar scheme works. However, as shown in (14), the network with constant channels loses randomness by only applying the naive fluctuation coding, so that normal transmission is not available as shown

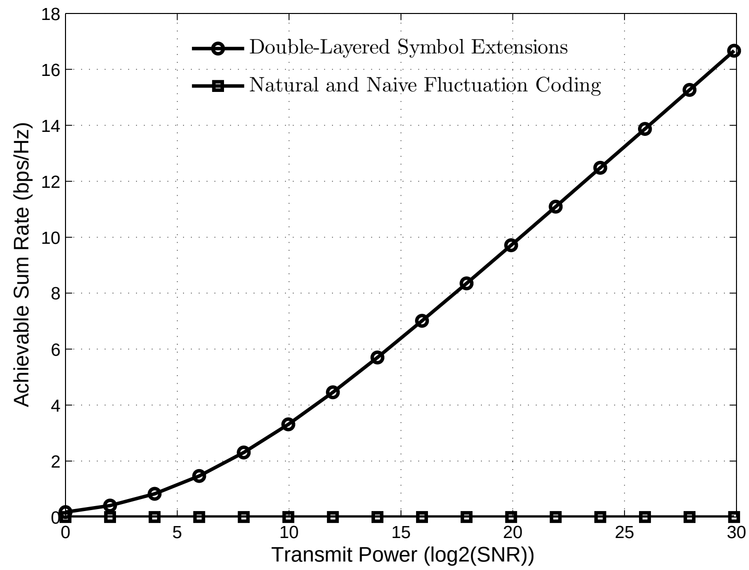

Fig. 1. Sum Rate of a 3-User Network Applying IA with Symbol Extensions in Constant Channels

in Fig. 1. For the double-layered symbol extensions, set the length of symbol extensions $T = 2(2n+1)$ and $n = 2$, so that the network is supposed to obtain a total DoF of $\frac{3n+1}{2(2n+1)} = 7/10$ with an additional $1/2$ factor due to the double-layered extension. As shown in (18), the network successfully creates virtual time-variant channels to apply effective interference alignment to approach $K/4$ DoF, so that it is clearly shown in Fig. 1 that the network obtains $7/10$ DoF when $n = 2$.

Theoretically, when $K = 4$, the expected achievable DoF of the network could approach $K/4 = 1$; when $K = 5$, the expected achievable DoF of the network could approach $K/4 = 1.25$. So that the achievable DoF could surmount the previous obtained DoF of 1 and 1.2 in [2] and [4] respectively. However, we are not able to illustrate the numerical results due to limited computational capability. Set $K = 5$ and $N = (K-1)(K-2)-1 = 11$. When $n = 81$, the total DoF is $\frac{(n+1)^{11}+4n^{11}}{2[(n+1)^{11}+n^{11}]} = 1.1995$; When $n = 82$, the total DoF is $\frac{(n+1)^{11}+4n^{11}}{2[(n+1)^{11}+n^{11}]} = 1.2001$. So in the case of $n = 82$, the DoF could surmount previous result of 1.2. However, at this time, notice $(n+1)^{11} = 1.2878e+021$, $n^{11} = 1.1271e+021$, and $(n+1)^{11} + n^{11} = 2.4149e+021$, so that $\mathbf{H}_{kj} \in \mathbb{C}^{(2.4149e+021)\times(2.4149e+021)}$,

$\mathbf{V}_1 \in \mathbb{C}^{(2.4149e+021)\times(1.2878e+021)}$ and

$\mathbf{V}_j \in \mathbb{C}^{(2.4149e+021)\times(1.1271e+021)}, \forall j \in \mathcal{K}\backslash\{1\}$. The super large dimensionality makes it difficult to shown numerical results.

V. Conclusion

In this work, we propose a novel method of double-layered symbol extensions to generate virtual time-variant channels to apply conventional Cadambe-Jafar scheme of interference alignment.

References

[1] V. Cadambe and S. Jafar, “Interference alignment and degrees of freedom of the K-user interference channel,” *IEEE Transactions on Information Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[2] A. Høst-Madsen and A. Nosratinia, “The multiplexing gain of wireless networks,” in *International Symposium on Information Theory*, Sept. 2005, pp. 2065–2069.

[3] S. W. Choi, S. A. Jafar, and S.-Y. Chung, “On the beamforming design for efficient interference alignment,” *IEEE Communications Letters*, vol. 13, no. 11, pp. 847–649, November 2009.

[4] V. Cadambe, S. A. Jafar, and C. Wang, “Interference alignment with asymmetric complex signaling - settling the Høst-Madsen-Nosratinia conjecture,” *IEEE Transactions on Information Theory*, vol. 56, no. 9, pp. 4552–4565, September 2010.